TREATISE

OF THE

COMBINATIONS, } { PERMUTATIONS

&

ELECTIONS, } { COMPOSITION

OF

QUANTITIES.

ILLUSTRATED

By several Examples, with a New Speculation of the Differences of the Powers

OF

NUMBERS.

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TO THE READER.

Courteous Reader,

A*Bove a Year since, on the Entreaty of a very Worthy and Publick-Spirited Friend, I gave my Consent that these Papers should come to light; afterward understanding that some* French *Authors had writ on this Subject, I put a stop to the Press; at length having obtained those Authors, and perused what they have said concerning the Argument herein handled, was willing (to prevent any abuses) that the Press should proceed. I have since out of those Books added two things; the first out of* Malbranch, alias *the Author of the* Elemens des Mathematiques; *namely, t give the number of the several Compositions that may be made of* 24 *Letters, and that on a double account, one to correct a Mistake of the* Printer *of that Treatise; for I do suppose it to be no other, (for that the Number of Figures, as also the first and the last are*

 true;)

true;) the other, to shew the manner of Operation, which he hath omitted. The Second is a Demonstration out of Monsieur Pascal's Tract du Triangle Arithmetique of what I had before by chance found out. Which you will find in Page 33, but misplaced; for it should follow Page 42.

In one place I have made use of Logarithms to calculate Numbers consisting of 30 Figures, which possibly may seem strange to some, for that the Table of Logarithms which I used, doth not promise to give Logarithms beyond 5 places: To which Doubt I answer, that exactly they cannot work beyond 5 places, but if they consist of 30 or more Figures, the first 4 Figures with the Number of Places will be true (as may easily be demonstrated) which is enough to make good my Proposition.

Farewel.

Of the Several Combinations, Elections, Variations and Compositions of Quantities.

1. BY Combination of Quantities, I mean how many ſeveral ways I may take any Number of Quantities out of any given Number of Quantities, without having any reſpect to their places; as how often out of the Engliſh Alphabet, I may take 2, 3, or more Letters; as two Letters, *ab*, *ac*, *cd*, *fg*, *fn*, &c. or three Letters, *abc*, *cfl*, *bmg*, *hnp*, *cde*.

2. By Election of Quantities, I mean (as *Francis Schoten* in his *Miſcellanies*) any Number of Quantities being given, how many ſeveral ways I may take them without having reſpect to their places; as, *a*, *b*, *c*, may be taken ſeven ways; as, *a*, *b*, *c*, *ab*, *ac*, *bc*, and *abc*.

3. By Variations, permutation or changes of the Places of Quantities, I mean, how many ſeveral ways any given Number of Quantities may be changed, as in relation to their places: as, *a*, *b*, may be changed into *b*, *a*; *a*, *b*, *c*, ſix ways; *abc*, *acb*, *bac*, *bca*, *cab*, and *cba*.

4. By Compoſition of Quantities, which is the moſt compoſed way, I mean, when in any Number of given Quantities, as in Letters or Figures, one Row is joyned with another Row of the ſame, or with 2, 3 or more other Rows; as is to be ſeen in the Tables of Natural Numbers, the placing of Letters, and the chances of the Dice. This differs from Combination and

Glen Rec 11 May 42 Henley

Election of Quantities; for that there one Quantity is taken but once, here, as often as there are Quantities to be taken.

Of the Combination of Quantities.

5. As out of Ten Letters, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, or the Ten Figures, how many Combinations are there of Two Letters or Figures?

There is with *a*, nine Combinations, *viz.* *ab*, *ac*, *ad*, *ae*, *af*, *ag*, *ah*, *ai*, and *ak*: with *b*, 8 Combinations, besides *ba*, already accounted, *viz.* *bc*, *bd*, *be*, *bf*, *bg*, *bh*, *bi*, *bk*; with *c*, 7 Combinations, besides *ac*, *bc*; with *d*, 6 Combinations; with *e*, 5; with *f*, 4; with *g*, 3; with *h*, 2; with *i*, 1.

Their Number by the Table of Figurate Numbers, may easily be found thus;

Take the number of Letters used in the Combination, lessened by an Unit, out of the Number given, and with that enter the second Row of the Table of Figurate Numbers, and even with it in the Row belonging to the Combination, is the Number sought; as $10-1=9$; with it enter the Table, and in the third Row (there being but 2 Letters in the Combination) is 45, and so many several Combinations there are of 2 Letters in 10.

6. How many several Combinations are there of 3 Letters in ten? You must take 2 out of (the Number given) Ten, and with the remainder 8, enter the second Row of the Table, and the Num. in the 4th. Row even with it, is 120; and so many several Combinations there are of 3 Letters in 10. The reason why 2 is taken from the given Number, is, for that the two first Letters do not vary the acceptation; as, *a b c*, *a b d*, *a b e*, *a b f*, *a b g*, *a b h*, *a b i*, and *a b k*. So that here are but eight several Combinations with *a b*. On the same account, if there be taken four Letters, there will be seven diverse Combinations with *a b c*; if five Letters, six Combinations with *a*, *b*, *c*, *d*.

7. How

7. How many ſeveral Combinations are there of four Quantities in Ten? 10—3=7; with which enter the ſecond Row of the Table, and even with it in the fifth Row, is 210; and ſo many ſeveral Combinations there are of Four Quantities in Ten; and proceeding after the ſame method, you will find there are 210 Combinations of ſix Quantities, 120 of ſeven Quantities, 252 Combinations of five Quantities, 45 of eight Quantities, 10 of nine Quantities, 1 of ten in ten Quantities.

8. If it be required to find how many ſeveral Combinations there are of two Letters in the Engliſh Alphabet; I ſay, 24—1=23; with which I enter the ſecond Row of the Table, and even with it in the third Row ſtands 276, and ſo many Combinations there are of 2 Letters in the Alphabet. If more Combinations are ſought after, as of 3, 4, 5, 6, or 7 Letters; then 24—2=22; even with it in the fourth Row ſtands 2024, the number of Combinations of 3 Letters in the Alphabet; over againſt 21 in the fifth Row, you will find 10626, the number of Combinations of 4 Letters; after the ſame manner, you will find 42504 Combinations of five Letters; 134596 of 6 Letters; 346104, of 7 Letters; 735461 of 8 Letters in the Alphabet.

9. But in caſe of Combinations, where there is no figurate Table, or the number of Quantities do exceed the Table; then thus.

Firſt, Place the number of Quantities given, then decreaſe that number by an Unite, as often as is the number of Quantities in the Combination, which place following one the other, with the Sign of Multiplication, ×, between them as the Dividend, and then place an Unite with the like number of Figures increaſing by an Unite, with the Sign of Multiplication ×, between them as a Diviſor.

Then (inſtead of multiplying thoſe Numbers according to their Signs for a Dividend and a Diviſor, as *Tacquet* teaches) prepare the Terms, by dividing the Diviſor and Dividend by each

each several number in the Divisor, then the Divisor will be brought to an Unite; then multiply the remainder of your Dividend, and you have your desire; as in the Example given *Sect.* 5, 6, 7. Where it is desired to know how many Combinations there are of 2, 3, 4, 5, 6, 7, 8, 9, and 10 Quantities in Ten.

$$\begin{array}{l} 2\times 1 \\ 1 \end{array}\Bigg) \begin{array}{l} 10\times 9\ (45 \\ 5 \end{array}$$

Place 10 and 9 as the Dividend, 1 and 2 as the Divisor, with the Sign of Multiplication × between them; then divide 2 in the Divisor, and 10 in the Dividend by 2, the quotients are 1 and 5; which multiplied in 9, the Product 45 is the number of Combinations of 2 quantities in 10.

If you would know how many Combinations there are of three Quantities in ten, add 8 to the Dividend, and 3 to the Divisor, with the Sign of Multiplication ×; thus,

$$\begin{array}{l} 3\times 2\times 1 \\ 1\times 1 \end{array}\Bigg) \begin{array}{l} 10\times 9\times 8\ (5\times 3\times 8=120. \\ 5\ \times 3 \end{array}$$

Afterwards prepare the Terms by dividing 9 in the Dividend, and 3 in the Divisor, by 3, then your Quotient is $5\times 3\times 8=120$, the number of Combinations of three Quantities in ten. If you desire to know the number of Combinations of four Quantities in ten, then add 7 to the Dividend, and 1 to the Divisor,

$$\begin{array}{l} 4\times 3\times 2\times 1 \\ 1\times 1\times 1 \end{array}\Bigg) \begin{array}{l} 10\times 9\times 8\times 7(5\times 3\times 2\times 7=210 \\ 5\ \times 3\times 2 \end{array}$$

and divide 4 in the Divisor, and 8 in the Dividend, by 4, the Quotes are 1 and 2: then $5\times 3\times 2\times 7=210$ the Combinations of four Quantities in ten.

If more Combinations are sought, proceed as before;

$$\begin{array}{l} 9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1 \\ 1\times 1\times 1\times 1\times 1\times 1\times 1\times 1\times 1 \end{array}\Bigg) \begin{array}{l} 10\times 9\times 8\times 7\times 6\times 5\times 4\times 3\times 2(\\ 5\ \times 3\times 2 \end{array}$$

The

The Factors of the Quotes.		*The Quotes.*		*Letters.*	
5×9		45		2	
5×3×8		120		3	
5×3×2×7		210		4	
3×2×7×6	=	252	of	5	in Ten.
3×2×7×5		210		6	
3×2×5×4		120		7	
3×5×3		45		8	
5×2		10		9	

The Demonstration hereof I defer to the last Sheet.

For the finding several Combinations in the English Alphabet; thus.

$$\frac{\text{Dividend, } 24\times23\times22\times21\times20\times19\times18\times17.}{\text{Divisor, } 8\times7\times6\times54\times3\times2\times1.}$$

The Factors of the Quotient.		*The Quot.*		*Letters.*
12×23		276		2
4×23×22		2024		3
23×22×21		10626		4 in the
23×22×21×4	=	42504	of	5 Alpha-
23×22×7×2×19		134596		6 bet.
23×22×2×19×16		346104		7
23×11×19×9×17		735471		8

Where there are several Combinations sought, it is best not to multiply the Factors that are in the Quotient together, until they are all found; for there are some Factors, which are common to most of the Quotients; as here $23\times22=506$ is common to the 5th middle Quotient.

10. It is desired to know how many several Lifts there are of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 Cards in a Pack.

For as much as 52, the number of Cards in a Pack, do exceed my Table of Figurate Num. it may be resolved this way.

$$12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1\ \Big)\ \begin{matrix}52\times51\times50\times49\times48\times47\\ \times46\times45\times44\times43\times42\times41.\end{matrix}\ \Big($$

The

The Factors of the Quotient.		*The Quotients.*		*The several Lifts*	
2, 26×51		1326		2	
3, 26×17×50		22100		3	
4, 26×17×25×49		270725		4	
5, 13×17×5×49×48		2598960		5	
6, 13×17×5×49×8×47	=	20358520		6	
7, 13×17×5×7×8×47×46		133784560	of	7	*Cards in a Pack.*
8, 13×17×5×7×47×46×45		752438150		8	
9, 13×17×5×7×47×46×5×44		3679075400		9	
10, 13×17×5×7×47×46×22×43		15820024220		10	
11, 13×17×5×7×47×46×2×43×42		60403728840		11	
12, 13×17×5×7×47×46×43×7×41		206379406870		12	

Here $13 \times 17 = 221$ is common to every *Quotient.*

For if you multiply it in 6, you have the first Quotient 1326, if in 100, the second, 22100.

Again, $13 \times 17 \times 5 \times 7 \times 47$ is common to the seven last Quotients.

So that if 1000 men shall constantly deal 12 Cards for 12 hours each day, excepting *Sundays*, they cannot deal all the several Games that are on 12 Cards in a Pack, in 54 Years, accounting that each man may deal a thousand Games in an hour.

11. How many several Stocks may there be at *Gleek?*

Forasmmuch as *Gleek* is played but with 44 Cards, and there are 7 Cards in the Stock, besides the Card turned up *Trump*, we must enquire how many Combinations there are of 7 in 44.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)\,44 \times 43 \times 42 \times 41 \times 40 \times 39 \times 38\,(\,44 \times 13 \times 41 \times$$
$$13 \qquad 13 \times 38.$$

=38320568, the Combinations of 7 Quantities in 44, which multiplied in 37; for that each Stock may have either of the remaining Cards, which are 37, turned up *Trump*, produces 1417861016, the number of the Several Stocks that may be at *Gleek.*

12. An old Captain having spent most part of his Time and Estate in the Wars, at the end thereof, desired of the King whom he served, as a recompence of his Service, a Farthing for

for each feveral File, containing fix Souldiers, that he could make in his Company, which confifted of 100 Souldiers: The King confidering his Requeft, and the greatnefs of his Defert, granted it. How much did it come to? *Anfw.* 1241721 *l.* 5 *s.*

$$6\times5\times4\times3\times2\times1)\,100\times99\times98\times97\times96\times95\,\Big(\begin{array}{l}10\times33\times98\times97\times4\times95\\=1192052400;\end{array}$$

or, $100\times33\times49\times97\times4\times19=1192052400$, the number of Files or feveral Combinations of 6 in 100 Quantities, which being divided by 960, the Farthings in a Pound; the Quotient is 1241721 *l.*, 25.

13. *Sempronius* bought of *Caius* 50 Sheep, to be drawn forth of 100, and being fomewhat tedious in choofing of them, *Caius* faid to *Sempronius*, give me one Barley-Corn for each feveral parcel of 50 Sheep that may be drawn forth of this 100, and you fhall have the whole 100. I will, anfwered *Sempronius*; and gave him 5 *l.* in Earneft: How much doth the value of the Sheep come to?

Anfw. That if the Terreftrial Globe fhould be covered with *Guineys* ten foot thick, there would not be enough to pay for the Sheep: Neither if the Terreftrial Globe fhould be converted to Barley, would there be Barley enough to fatisfie for the Sheep. Which I thus demonftrate,

Proceeding according to the foregoing Rules, you will find the Factors in the Quotients to be,

$97\times19\times93\times13\times89\times87\times17\times83\times9\times79\times11\times73\times71\times67\times61\times59\times53\times8$; which if you work with Logarithms, (which will be exact enough) will amount unto 10088 with 25 Cyphers, whofe Logar. is, 29, 003862 for the Barley-Corns to be paid for the Sheep, I have exactly weighed an Ounce *Aver-du-pois* of Barley, and it contained 681 Grains; and therefore one Pound contains 10896 Grains, and a Bufhel 544800 Gr. For I found that a Bufhel of the fame Barley containing eight Gallons, did weigh 50 *l. aver-du-pois*, whofe Logar. 5,736236 being

being deducted from 29.003862. remains 23,167416, whose Number is 1851, with 20 Cyphers, for the Bushels of Barley to be paid for the Sheep.

Which accounting at 2 *s.* the Bushel, comes to 1851, with 19 Cyphers, for the worth of the Barley in Pounds.

The Circuit of the Earth, by the most common account, is, 21600 Miles; its Logarithm, 4,334452.

Which squared, is,	466560000	8,668908.
To it add		1,502850.
Then you will have	148411000.	8,171758.
The Superficies of the Earth in square Miles.		
Every Mile is in length 5280 Feet.	Logar.	3,722634
Which squared, is	27878400	7,445264
Which added to the Log. of the square Miles,		8,171758
renders 414021, with ten Cyphers		15,617022

Equal to the Number of square Feet, on the Surface of the Globe.

The Cubical inches in a Cubick Foot,	1728. Log.	3,237544
Multiplied in the Ounces that are in a Cubical Inch of Gold, (*Mathematical Compendium*, p. 17.)	9,91735.	0,996396
		4,233940

You have the Log. of the Ounces contained in a Cubical Foot of Gold; which divided by the Ounces in a ℔. 12. is 1,079181

You have the *Troy* Pounds in a Cubick Foot of Gold	1428.	3,154759
Which multiplied by the value of 1 ℔. *Troy* of Gold,	40,918.	1,611913
You have the value of a Cubick Foot of Gold.	58435, 1.	4,766672
Which multiplied in the number of Feet, with ten Cyphers.	414025,	15.617022
		20,383694.

You

You have 2419, with 17 Cyphers, for the value of the Gold that covers the Earth one Foot.		20,383694
Which deduced from the Log. of the value of the Barley		22,267426 20,383694
Whose Number	74,77	1,873732

shews, That if the whole Surface of the Earth was covered with Gold 74 feet thick, it was but the price of the Barley. *q. e d.*

Again, to the Cube of the Circuit of the Earth,	10077696000000	13,003362
Add		2,227549
You have the solid Content of the Globe in Cubick Miles.	170182000000;	11,230911
Which multiplied into the Cube of 5280, makes the Cubick Feet in a Cubick Mile,	147197952000	11,167902
You have 2505 with 19 Cyphers, = to the Cubick Feet contained in the Globe.		22,398813

According to Mr. *Gunter* and Mr. *Oughtred*'s Experiments, 231 cubick inches are in a Gallon; therefore 1848 cubick inches are in 8 Gallons or a Bushel.

As 1848,3,266702, The Cubical inches in a Bushel.
is to 1728, ,237544, The Cubical Inches in a Cubical Foot.
So is 544800.5,736236, The Grains in a Bushel by trial,

to 509422.5,707078. The Grains in a Cubical Foot, Multiplied in 22,398813. The Cubical Feet in the Globe 1276

*25 cyphers, 28,105891, shews how many Barley-Corns if the

Globe

Globe was converted to Barley, it would contain; which substracted from the Log. 29,003862, the price of the Sheep.

28,105891

7,906. 0,897971, shews that above 7 such Worlds, if converted to Barley, is not enough to pay for the Sheep: *q. e. d.*

Or if the whole Globe was covered 3436 Miles, equal to the Semidiameter of the Earth high in Barley, there would not be enough.

For that would be equal to 7 times this Globe, whose Semidiameter is 3436 Miles; Spheres being in triplicate proportion of their Diameters 12 *E.* 8. that is here where the Diameters are as, 1 and 2, the Spheres will be as 1 and 8, take the difference of the Spheres, it will be as 1 to 7.

Of Election of Quantities.

13. The Election of Quantities may likewise be found out by the Table of Figurate Numbers, or else by the foregoing Rule; for it is but the addition of the several Combinations that are under it to the Number it self.

As, How many are the Elections of ten Quantities?

By the Table I must see how many Combinations there are of 2, 3, 4, 5, 6, 7, 8, 9, and 10, in ten Quantities, as is already done in the 5th, 6th, 7th, and 9th. Section, which added to 10, makes 1023, the Elections, thus;

$10+45+120+210+252+210+120+45+10+1=1023$, the Elections of 10 Quantities.

14. How many Conjunctions may there be of the seven Planets?

The Election of 7 Quantities lessened by 7, the number of Planets is equal to the Conjunction of the Planets; as, $21+35+35+21+7+1=120$, the Conjunctions, that is to say, there are 21 Conjunctions of 2 Planets, 35 Conjunctions of 3 Planets,

35

35 of 4 Planets, 21 of 5 Planets, 7 of 6 Planets, and 1 of 7 Planets.

15. There is a Key that hath 8 ſeveral Wards, and it is deſired to know, how many Locks whoſe Wards differ, may be made which that Key may unlock? I anſwer, 255 Locks.

1 2 3 4 5 6 7 8

8+28+56+70+56+28+8+1=255 ſeveral Locks; 8 whereof, have but one ſingle Ward, 28 have double Wards, 56 treble Wards, 70, 4 Wards, 56, 5 Wards, 28, 6 Wards, 8, 7 Wards, and 1 Lock hath 8 Wards.

But if it had been required to know how many ſeveral Locks might have been made, which that Key might unlock, and neither of the other Keys? I Anſwer, 70 Locks; and each Lock muſt have 4 ſeveral Wards.

16. But in caſe of Elections, where it is deſired to have it performed at one Operation;

Then it may be done by the Rules of Geometrical Progreſſion.

Let $r=$ to the *Ratio*, which here is always 2.

$s=$ the Sum of the *Terms*.

$t=$ the number of the *Terms*. Then

$r^{(t)}-1=s$. That is to ſay, Multiply the *Ratio* 2, as often in it ſelf, as is the Number of *Terms*, and from it ſubſtract 1. the Remainer is the Sum of the Elections.

As the 10*th* power of 2 is 1024. from which ſubſtract 1. the Remainer ſhews its Elections.

17. How many Elections are there of the Letter of the Alphabet? $r(t)-1$. *i.e.* $r^{(24)}-1$.

$r^{(24)}=16777216=$ to the 24*th* power of 2.

Subſtract 1.

16777215. The Number of Elections of 24 Letters.

 That

That Elections do proceed as the Sums in Geometrical Progression, whose *Ratio* is 2. appears; as *Francis Schoten* in his *Miscellanies* hath demonstrated: as take 4 several Magnitudes, *a, b, c, d,* in order.

Sum	*Elect.*	*Dif.*
a.	1	1.
a, b. ab.	3	2.
a, b, c, ab, ac, bc, abc.	7	4.
a, b, c, d. ab, ac, ad, bc, bd, cd, abd, abc, acd, bcd, abcd.	15	8.

Or thus, leaving out those that are mentioned in the foregoing Election.

a.	1.
b, ab.	2.
c, ac, bc, abc.	4.
d, ad, bd, abd, cd, acd, bcd, abcd.	8.

So that these which are the Differences of the Elections, do all proceed in Geometrical Progression, and the Elections are the Sums of Quantities proceeding in Geometrical Progression.

	Elect.	Dif. of Elections.
a.	1.	1.
ab.	3.	2.
abc.	7.	4.
abcd.	15.	8.
abcde.	31.	16.
abcdef.	63.	32.
abcdefg.	127.	64.
abcdefgh.	255.	128.
abcdefghi.	511.	256.
abcdefghik.	1023.	512.
abcdefghikl.	2047.	1024.
abcdefghiklm.	4095.	2048.
abcdefghiklmn.	8191.	4096.
abcdefghiklmno.	16383.	8192.
abcdefghiklmnop.	32767.	16384.
abcdefghiklmnopq.	65535.	32768.

Of

Of Variations of Quantities.

18. Having considered the Combinations and Election of Quantities, let us consider their Variations or Changes of Places.

One Quantity can admit of no Variation.

Two Quantities admit of 2 Variations; as *a b*, *b a*.

Three Quantities of 6 Changes; as, *a b c*, *a c b*, *b a c*, *b c a*, *c a b* and *c b a*; which is found by Multiplying their Indexes together, *viz.* into ; for two Quantities or Letters, admitting of 2 Changes, there are 3 several times Letters; and consequently 6 Changes; as, *a*, *b*, *c*, do admit of 3 Combinations by 5 Sect. *viz.* *a b*, *a c*, *b c*: and each of these of 2 Variations; therefore the Variations of 3 Quantities are 6.

Four Quantities hath 24 Changes; for 3 Letters admitting of 6, there are four several times 3 Letters; and consequently 4 times 6 Changes, *i. e.* 24; as, *a*, *b*, *c*, *d*: here is *a b c*, *a b d*, *a c d*, and *b c d*. Now each of these, as by the last, do admit of 6 Changes: To the Changes of *a b c*, prefix *d*, then you have 6 Changes; *d a b c*, *d a c b*, *d b a c*; *d b c a*, *d c a b*, *d c b a*. So to the 6 Changes of *a b d*, prefix *c*, you will have 6 other Changes. So to the 6 Changes *a c d*, prefix *b*, you will have other 6 Changes. And again, to the 6 Changes of *b c d*, prefix *a*, you will have 6 other Changes. So in all you have 24 Changes of *a*, *b*, *c*, *d*.

Five Letters, on the same account, admits of 120, made by Multiplication of 24 in 5.

Six Letters of 720, made by Multiplication of 120 in 6.

Seven Letters of 5040, made by Multiplying 720 in 7.

19. It is required to know how many several Combinations with its Variations, there are of 3 Letters in the English Alphabet. Multiply its Combinations 2024, in its Changes 6, and

and the Product 12144 is the Number of placing 3 Letters in the Alphabet, with all their Variations.

20. But if there are 2, or more Quantities or Letters of one sort, then divide the whole Number of Changes by the Changes of the Number of those Letters, and the Quotient is the Number of Changes; as, *a, a, b*. Divide 6 its Changes by 2, the Changes of 2 Letters, the Quotient is 3, and so many Changes are there.

So *a a a b c* hath 20 Changes; for Divide its Changes 120, by 6, the Changes of 3, its Quotient is 20.

If there are 2, or more Letters that have 2 or more Letters of one sort, then Multiply their Changes together for a Divisor; as, *a a a b b*, Divide its Changes 120 by 12, the Product of 6 in 2, being the Changes of 3 and 2, the Quote is 10, its Changes.

So there are 6 Changes of *a a b b*, 210 Changes of *aaabbcc*.

Quantity.	*Variations.*
1.	1.
2.	2.
3.	6.
4.	24.
5.	120.
6.	720.
7.	5040.
8.	40320.
9.	36288.
10.	3628800.
11.	39916800.
12.	479001600.
13.	6227020800.
14.	87178291200.
15.	1307674368000.
16.	20922789888000.
17.	355687537996000.

18.

18. 6402375683928000.
19. 121645137994632000.
20. 2432902759892640000.
21. 51090957957745440000.
22. 1124001075070399680000.
23. 25852024726619192610000.
24. 620448593438860623360000.

By this Method may be found how many ſeveral Changes there may be on each Chance on any Number of Dice; as,

On 3 Dice.

Chances.	*Changes.*
1, 1, 1.	1.
1, 1, 2.	3.
1, 2, 3.	6.

On 4 Dice.

Chances.	*Changes.*
1, 1, 1, 1.	1.
1, 1, 1, 2.	4.
1, 1, 2, 2.	6.
1, 1, 2, 3.	12.
1, 2, 3, 4.	24.

On 6 Dice.

Chances.	*Changes.*
1, 1, 1, 1, 1, 1.	1.
1, 1, 1, 1, 1, 2.	6.
1, 1, 1, 1, 2, 2.	15.
1, 1, 1, 1, 2, 3.	30.
1, 1, 1, 2, 2, 2.	20.
1, 1, 1, 2, 2, 3.	60.
1, 1, 1, 2, 3. 4.	120.
1, 1, 2, 2, 3, 3.	90.
1, 1, 2, 2, 3, 4.	180.
1, 1, 2, 3, 4, 5.	360.
1, 2, 3, 4, 5, 6.	720.

21. How many Elections are there of 10 Figures with all its Variations?

Multiply all its ſeveral Combinations with its proper Changes, then the Sum of the Products is equal to all the Elections with their Changes.

Multiply

	10 in 1	10.
	45 in 2	90.
	120 in 6	720.
	210 in 24	5040.
Multiply	252 in 120	= 30240.
	210 in 720	151200.
	120 in 5040	604800.
	45 in 40320	1814400.
	10 in 362880	2628800.
	1 in 3628800	2628800.
		9864100 = all its Elections

with their several Changes.

Of Composition of Quantities.

Although this is the most composed way, yet this is the easiest to be performed; for if the composition of two Quantities in 10, or any other Number given (as the Alphabet) is sought; it is but squaring 10 or 24, and therefore there are 100 Conjunctions of 2 Figures in 10; 576 of 2 Letters in the Alphabet.

27. If the composition of 3 Quantities be sought, it is the cubing of the given Number, as, 1000, of 3 Figures. 13824 of the Letters in the Alphabet.

If the composition of 4 Quantities, then it is the Biquadratick power of the number; as 10000 of the Figures, 331776 of the Letters.

If of 5 Quantities, the Quadricubick power; as, 100000 of the Figures. 7962624 of the Letters.

If of 6 Quantities, the Cubocubick power; as, 1000000 of the Figures, 191102976 of the Letters; and so of more Quantities.

It may be Objected, that herein I am mistaken; for in the Table

Table of Numbers there are but 900 Conjunctions of 3 Figures; for the Figures under 10, are but of one place; between 10 and 100, of 2 places.

I Anſwer; If in writing thoſe numbers, you will ſupply what is to be underſtood, by filling up the void places with Cyphers, as 000, 001, 002, 003, 004, &c. 011, 012, 013, 021, 029, 075, &c. you will find there are 1000 Numbers that conſiſt of 3 places.

How many ſeveral diſpoſitions may there be of the 24 Letters, taking them by one and one, two and two, three and three, and ſo to 24?

If it had been demanded to know how many Diſpoſitions or Compoſitions there are of 24 Letters, accounting each time 24, the Anſwer would have been, 1333735776850284124449081472843776, The Number of ſeveral Compoſitions that 24 Letters may make, being the 24th. power of 24: But being we are to find all the Numbers proceeding in Geometrical Progreſſion under it,

Let $r =$ *Ratio* $= 24$, and $a =$ *firſt Term* $= 24$.

$t =$ Number of Terms,

$z =$ Sum of all the Terms.

$r - 1 \,.\, a :: r^{(t)} - 1 \,.\, z.$ (Dr. *Wallis* Arithm.)

that is to ſay, as the *Ratio* leſſen'd by an Unit, is to the *firſt Term*, ſo is the *Ratio* multiplied in it ſelf as often as is the number of *Terms* leſſened by an Unit, to the Sum of all the *Terms*;

The 24th. power of 24 is

1333735776850284124449081472843776. Which leſſened by an Unite, and encreaſed by $\frac{1}{23}$ of it, is the Num. deſired, thus;

$r^{(t)} - 1$.	1333735776850284124449081472843775.
$+ \frac{1}{23}$	57988512036968874976047012055842 5.
	1391724288887252999425128493402200. the

Number ſought. Which in a *French* Author, Entituled, *Elemens des Mathematiques*; Publiſhed 1675 is miſs-calculated, where he gives you the Anſwer, but not the manner of Operation.

24. How many several Chances are there on 2, 3, 4, 5 and 6 Dice?

According to the foregoing Rules, I say there are 36 chances on 2 Dice, 216 chances on 3 Dice, 1296 chances on 4 Dice, 7776 chances on 5 Dice, 46656 chances on 6 Dice.

A more particular Account according to the foregoing Rules of Variation, on 2, 3, and 4 Dice, followeth.

The several Chances on 2 Dice.

Casts.	*Points.*	*Changes.*	*Sum.*
2, 12.	1, 1.	1,	1.
3, 11.	1, 2.	2.	2.
4, 10.	1, 3. 2, 2.	2 1	3.
5, 9.	1, 4. 2, 3.	2. 2.	4.
6, 8.	1, 5. 2, 4. 3, 3.	2. 2. 1.	5.
7.	1, 6. 2, 5. 3, 4.	2. 2. 2.	6.

The Sum of the Chances of 2, 3, 4, 5, 6 Casts, are 15: So likewise of 12, 11, 10, 9, 8 Casts, are 15, which I have forborn to set down in particular, for that they are the Sum; as, 2, 3, 4, 5, 6. *Mutatis mutandis.*

To which add the Chances of 7, which are 6; their Sum is 36, the Chances of 2 Dice.

The Chances on 3 Dice.

Casts.	*Points.*	*Changes*	*Sum.*
3 18	1 1 1	1	1.
4 17	1 1 2	3	3.
5 16	1 2 2 1 1 3	3 3	6.
6 15	1 1 4 1 2 3 2 2 2	3 6 1	10.

7 14	1	1	5	3		15.
	1	2	4	6		
	1	3	3	3		
	2	2	3	3		
8 13	1	1	6	3		21.
	1	2	5	6		
	1	3	4	6		
	2	2	4	3		
	3	3	2	3		
9 12	1	2	6	6		25.
	1	3	5	6		
	1	4	4	3		
	2	3	4	6		
	2	2	5	3		
	3	3	3	1		
10 11	1	3	6	6		27.
	1	4	5	6		
	2	3	5	6		
	2	2	6	3		
	2	4	4	3		
	3	3	4	3		

The Casts.		*Sum of the Changes*
3	18	1.
4	17	3.
5	16	6.
6	15	10.
7	14	15.
8	13	21.
9	12	25.
10	11	27
		108.

Which Multiplied in 2 (for the Chances, of 18, 17, 16, 15, 14, 12, & 11, are so many) produces 216, the chances of 3 Dice.

The several Chances on 4 Dice.

Casts.		*Points.*				*Chan.*	*Sum.*
4	24	1	1	1	1	1	1.
5	23	1	1	1	2	4	4.
6	22	1	1	1	3	4	10.
		1	1	2	2	6	
7	21	1	1	1	4	4	20.
		1	1	2	3	12	
		1	2	2	2	4	
8	20	1	1	1	5	4	35.
		1	1	2	4	12	
		1	1	3	3	6	
		1	2	2	3	12	
		2	2	2	2	1	
9	29	1	1	1	6	4	56.
		1	1	2	5	12	
		1	1	3	4	12	
		1	2	2	4	12	
		1	2	3	3	12	
		2	2	2	3	3	

Casts.	Points.	Chan.	Sum.
10 18	1 1 2 6	12	80.
	1 1 3 5	12	
	1 1 4 4	6	
	1 2 3 4	24	
	1 2 2 5	12	
	1 3 3 3	4	
	2 2 3 3	6	
	2 2 2 4	4	
11 17	1 1 3 6	12	104.
	1 1 4 5	12	
	1 2 2 6	12	
	1 2 3 5	24	
	1 2 4 4	12	
	1 3 3 4	12	
	2 2 2 5	4	
	2 2 3 4	12	
	2 3 3 3	6	
12 16	1 1 4 6	12	125.
	1 1 5 5	6	
	1 2 3 6	24	
	1 2 4 5	24	
	1 3 3 5	12	
	1 3 4 4	12	
	2 2 2 6	4	
	2 2 4 4	6	
	2 3 3 4	12	
	3 3 3 3	1	

Casts	Points.	Chan.	Sum
13 15	1 1 5 6	12	140
	1 2 4 6	24	
	1 2 5 5	12	
	1 3 3 6	12	
	1 3 4 5	24	
	1 4 4 4	4	
	2 2 3 6	12	
	2 2 4 5	12	
	2 3 3 5	12	
	2 3 4 4	12	
	3 3 3 4	4	
14	1 1 6 6	6	146.
	1 1 5 6	24	
	1 3 4 6	24	
	1 3 5 5	12	
	1 4 4 5	12	
	2 2 4 6	12	
	2 2 5 5	6	
	2 3 3 6	12	
	2 3 4 5	24	
	2 4 4 4	4	
	3 3 3 5	4	
	3 3 4 4	6	

Casts.	Sum of the Chances.
4, 24	1
5, 23	4
6, 22	10
7, 21	20
8, 20	35
9, 19	56
10, 18	80
11, 17	104
12, 16	125
13, 15	140
	575.

Which being doubled, is 1150; to which add 146, the Chances on 14, the Sum is 1296, the Chances on 4 Dice.

If you will take an account of the Casts whose Changes are 8, 4, & 6, you will find there are 216 Chances of double pairs, or *In and In*, as it is usually call'd in the Play on 4 Dice that usually bears that Name.

If you reckon the Chances whose Changes are 3, 4, in which are no Pairs, you will find them to be 360; and so many Chances there are where there is no Pair, and consequently there are 720 Chances where there are single Pairs.

The said number of 360 Chances having no Pair, may likewise be found by 5, 5, thus $6-3=3$. with which enter the Table, and even with it in the Fourth Row, stands 15, its Combinations; which multiplied in its Changes 24, produces 360, as before.

I will give you another Mathematical Observation on Dice.

The particular Chances on 2 Dice are,

1, 1.	1, 2	1, 3	14	15	16
2, 1	22	23	24	25	26
3, 1	32	33	34	35	36
4, 1	42	43	44	45	46
51	52	53	54	55	56
61	*62*	*63*	*64*	*65*	*66*

Here is 36 Chances the Square of 6, as before, all the Chances of 6 are placed in the lowermost and furthermost Row, so that in the Square of 5, *i. e.* 25, there is no 6. So there are 25 Chances without 6, and 11 where there is a 6.

In the Square of 4, *i. e.* 16 Chances, there is neither 5 nor 6.

In the Square of 3, *i. e.* 9, there is neither 4, 5, nor 6.

In the Square of 2, *i. e.* 4, there is neither 3, 4, 5 nor 6. This may be applied to other Chances.

a,

1,	2,	3,	4,	5,	6.
Lat.	q,	c,	qq,	qc,	cc.
1,	1,	1,	1,	1,	1.
2,	4,	8,	16,	32,	64.
3,	9,	27,	81,	243,	729.
4,	16,	64,	256,	1024,	4096.
5,	25,	125,	625,	3125,	15625.
6,	36,	216,	1296,	7776,	46656.

This is a Table of the Powers of 6, and of the Numbers under it; and by it may be accounted how many ſeveral Chances there are on 2, 3, 4, 5, or 6 Dice, and how many there are where there is no 6, or neither 5 nor 6, or neither 4, 5, nor 6; or neither 3, 4, 5, nor 6. On 2 Dice we have done it already.

On 3 Dice, I ſay that there are 125 Chances where there is no 6, and 91, where there is a 6; for 216—125=91; that is to ſay, in the Cube of 5 there is no 6. In the Cube of 4, that is, 64, there is neither 5 nor 6; and the Remainer (it being ſubſtracted from the Cube of 6, *viz.* 216.) is 152, which are the Chances where there is a 5, or 6.

216—27=189, which are the Chances where there is a 4, 5, or 6, and the Cube 27, ſheweth that there are 27 Chances that have neither 4, 5, nor 6.

If it be demanded on how many Chances on 4 Dice, there is no 6, I ſay on 625, the Biquadrate of 5; and 1296—625=671; and in ſo many Chances there is a 6.

In 256 Chances there is neither 5 nor 6. 1296—256=1040, and ſo many Chances hath either a 5 or a 6.

In 81 Chances there is neither 4, 5, nor 6; 1296—81=1215; ſo many Chances hath either a 4, 5, or 6.

In 16 Chances there is neither 4, 5, nor 6. 1296—16=1280, and ſo many Chances hath either 3, 4, 5, or 6.

After the ſame manner, on 5 or 6 Dice, their Chances may be

be ſhewn; and what is ſaid here of *6*, is true of any othe ſingle Chance.

Foraſmuch as all Powers of Numbers do conſiſt of ſeveral Combinations of Differences, as will appear; I will preſent you with a View of the Tables of the Powers of Numbers, as in relation to their Differences; and how (having any Power with its Differences) to find any other Power and its Differences.

1. The Differences of Squares do proceed in Arithmetical Progreſſion, and its common Difference is 2.

2. If you ſubſtract the Difference of the Differences of any Power, ſo often as the Index of the Power is, you will find they do encreaſe in Arithmetical Progreſſion, and their common Difference is found by Multiplying all the Indexes under it together, and in its own Index.

Square.

c.	*b.*	*d. First Difference.*	*f. Common Differ.*
0	0		
1	1	1	
2	4	3	2.
3	9	5	2.
4	16	7	2.
5	25	9	2.
6	36	11	2.
7	49	13	2.
8	64	15	2.
9	81	17	2.
10	100	19	2.

Cube.

	b.	*d. First Difference.*	*f. Second Difference.*	*g. C. D.*
0	0			
1	1	1		
2	8	7	6	
3	27	19	12	6.
4	64	37	18	6.
5	125	61	24	6.
6	216	91	30	6.
7	343	127	36	6.
8	512	169	42	6.
9	729	217	48	6.
10	1000	271	54	6.

Its *Common Difference 6*, is made by Multiplying its *Index 3*, in 2.

b.

	b. *Biquadrate*, or *Quadriquadrate.*	*d.* *Its first Difference.*	*f.* *Second Difference.*	*g.* *Third Difference.*	*h.* *Common Difference.*
0	0				
1	1	1			
2	16	15	14		
3	81	65	50	36	
4	256	175	110	60	24
5	625	369	194	84	24
6	1296	671	302	108	24
7	2401	1105	434	132	24
8	4096	1695	590	156	24
9	6561	2465	770	180	24
10	10000	3439	974	204	24.

Its *Common Difference* 24, is made by Multiplying its Index 4, in 3 in 2.

	b. *Quadricube.*	*d.* *First Difference.*	*f.* *Second Difference.*	*g.* *Third Difference.*	*h.* *Fourth Difference.*	*k.* *Common Dif.*
0	0					
1	1	1				
2	32	31	30			
3	343	211	180	150		
4	1024	781	570	390	240	
5	3125	2101	1320	750	360	120
6	7776	4651	2550	1230	480	120
7	16807	9031	4380	1830	600	120
8	32768	15961	6930	2550	720	120
9	59049	26281	10320	3390	840	120
10	100000	40951	14670	4350	960	120.

Its *Common Difference* 220, is made by Multiplying its Index 5, in 4, in 3, in 2.

Q. ſtands for Square, as *Q* 5. that is the Square of 5.

C. for Cube. *QQ.* for the Quadriquadratick, or Biquadrate.

QC. for Quadricube.

Any Power with its Differences being given, to find any other Power and its Differences, let *b* ſtand for the Power given; *d*, for its firſt Difference; *f*, for its ſecond Difference; *g*, for its third Difference; *h*, for its fourth Difference; *k*, for its fifth Difference; *m*, for its ſixth Difference.

And if the Power be greater, no more Symbols.

With the Differences of the ſides of the Powers given and ſought, enter the Figurate Table, and to the Power given *b*, add ſo many *d's* as are in the firſt Row even with the Difference of the ſides; and ſo many *f's* as are in the ſecond Row, and ſo many *g's* as are in the third Row, ſo many *h's* as are in the fourth Row, ſo many *k's* as are in the fifth, and ſo many *m's* as are in the ſixth Row, where occaſion requires, and the Sum is the Power ſought.

If the Power ſought, be a Squre, take only the two firſt Differences, *d*, and *f*.

If a Cube; uſe only the three firſt Differences, *d*, *f*, *g*.

If a Biquadrate; the four firſt Differences, *d*, *f*, *g*, and *h*.

If a Quadricube; the Differences *d*, *f*, *g*, *h*, and *k*.

If a Cubicube; the Differences *d*, *f*, *g*, *h*, *k*, and *m*.

The Square of 5 is $25=b$, $d=9$, $f=2$. What is the Square of 9? $9-5=4$. With it enter the Figurate Table, then $b+4d+10f=Q.\ 9.$

$25+36+20=81.$

The Cube of 3 is 27. What is the Cube of 10? Here $b=3.\ d=19.\ f=12,\ g=6.\ 10-3=7.$ With it enter the Table.

$b+7d+28f+84g=1000=$ Cube of 10.

$27+133+336+504=1000.$

The

The QQ. of 4, is 256. What is QQ. of 10. Here $b=256$. $d=175$. $f=110$. $g=60$. $h=24$. $10-4=6$. With it enter the Figurate-Table. Then,

$b+6d+21f+56g+126h=QQ.10.$

$256+1050+2310+3360+3024=10000.$

The QC. of 4 is 1024. What is the QC. of 10. Here $b=1024$. $d=781$. $f=570$. $g=390$. $h=240$. $k=120$. $10-4=6$.

$b+6d+21f+56g+126h+25k=QC.10.$

$1024+4586+11970+21840+30240+30240=100000.$

As the Powers are thus found, so are their Differences; if it be d its first Difference that is sought, then let the first Row be joyned to f, the second to g, the third to h, the fourth to k.

If it be f, the second Difference, then let the first Row be joyned to g, the second to h, the third to k.

If it be g, the third Difference, then let the first Row be joyned to h, the second to k, &c.

As in the Example of the Cube; $b=27$. $d=19$. $f=12$. $g=6$. What the Cube of 10 is we have found, what are the Differences?

$d+7f+28g=$ the first difference of $10.=d.$

$19+84+168=271=d.$

$f+7g=$ second difference f.

$12+42=54.$

The Cube of 10 is 1000, what is the Cube of 20?

$b=1000$. $d=271$. $f=54$. $g=6$. $20-10=10$. With 10 enter the Figurate Table.

$b+10d+55f+220g=$ Cube of 20.

$1000+2710+2970+1320=8000.$

$d+10f+55g=d$ first difference.

$271+540+330=1141.$

$f+10g=f$ second difference.
$54+60=114.$
$b+d+f+g=$ Cube of 21.
$8000+1141+114+6=9261.$

In the Example of the Quadricube given, what are the Differences?

$d+6f+21g+56h+126k=d.$
$781+3420+8190+13440+15120=40951.$

$f+6g+21h+56k=f$
$570+2340+5040+6720=14670$

$g+6h+21k=g$
$390+1440+2520=4350.$

$h+6k.=:$
$240+720=960$

$b+d+f+g+h+k=QC.11$
$100000+40951+14670+4350+660+120=161051.$

My manner of discovery hereof, was thus;

I first made these Observations on the Tables of the Differences of Powers.

1. Every Power is equal to its own Difference, and to all the Differences above it; as,

$Q\ 16=7+5+3+1.$ *Cub.* $64=37+19+7+1.$
$QQ.256=175+65+15+1.$ $QC.243=211+31+1.$

2. Every Power increased with all the Differences that stand in the same Row, is equal unto the next Power following; as,

$Q.16+7+2=Q.25.$
Cub. $64+37+18+6=C.125.$
$QQ.256+175+110+60+24.$ $QQ.625.$

3. Every

3. Every Power increased with the Difference under its own Difference, until you come even with any Power desired, is equal unto that Power; as, $Q.\ 9+7+9+11=Q.\ 36.$ $C.\ 8+19+37=C.64.$ $QQ.\ 16+65+175=QQ.\ 256.$

4. Every Difference is equal unto the Difference above it, and all standing even with that to the right hand, that is to be understood where the Table hath its full Differences.

As in the Table of Cubes, $91=61+24+6.$ $127=91+30+6.$

In the Table of Quadriquadrates, $671=369+294+84+24.$

In the next place, I began to examine the Tables thus; the Square of 2 is 4, what is the Square of 5? Here $d=3.f=2.$

3 *Observ.* $b+5+7+9=25=Q.\ 5.$
that is,

4 *Observ.*

$$\begin{array}{llll}
b+5\ i.e.\ 3+2. & d+f. & d+f. & 5. \\
+7\ i.e.\ 5+2\ i.e.\ 5.\ d+f. & & & \\
 & \left.\begin{array}{r} \\ 2\quad f. \end{array}\right\} & d+2f. & 7. \\
+9\ i.e.\ 7+2\ i.e.7.\ d+2f. & & d+3f. & 9. \\
 & \left.\begin{array}{r} \\ 2.\quad f. \end{array}\right\} & 3d+6f. &
\end{array}$$

So that $b+3d+6f=Q.5.$
$4+9+12=25.$

In the next place, I tried the Cube before mentioned thus;

$b=27.d=19.f=12.\ g=6.$ $\left\{\begin{array}{l} b+37+61+91+127+ \\ +169+217+271=1000. \end{array}\right.$

3 *Observ.* $b+37.\ i.e.\ 19+12+6.\ d+f+g.\ d+f+g=37.$

4 *Obs.*

$$4\ \textit{Observ.}\ +61.\ i.e.\ 37+18+6\ i.e.\ \left.\begin{array}{lr}37.\,d+f+g. & \\ 18. & f+g. \\ 6. & g.\end{array}\right\}\begin{array}{l}d+2f\\ +3g\\ =61.\end{array}$$

$$+91\ i.e.\ 61+24+6.\ i.e.\ \left.\begin{array}{lr}61.\,d+2f+3g. & \\ 24. & f+2g. \\ 6. & g.\end{array}\right\}\begin{array}{l}d+3f\\ +6g\\ =91.\end{array}$$

$$+127.\ i.e.\ 91+30+6\ i.e.\ \left.\begin{array}{lr}91.\,d+3f+6g. & \\ 30. & f+3g. \\ 6. & g.\end{array}\right\}\begin{array}{l}d+4f\\ +10g\\ =127.\end{array}$$

$$+169.\ i.e.\ 127+36+6\ \ \left.\begin{array}{lr}127.\,d+4f+10g. & \\ 36. & f+4g. \\ 6. & g.\end{array}\right\}\begin{array}{l}d+5f+\\ 15g=\\ 169.\end{array}$$

$$+217.\ i.e.\ 169+42+6\ i.e.\ \left.\begin{array}{lr}169.\,d+5f+15g. & \\ 42. & f+5g. \\ 6. & g.\end{array}\right\}\begin{array}{l}d+6f\\ +21g\\ =217\end{array}$$

$$+271.\ i.e.\ 217+48+6.\ i.e.\ \left.\begin{array}{lr}217.\,d+6f+21g. & \\ 48. & f+6g. \\ 6. & g.\end{array}\right\}\begin{array}{l}d+7f\\ +28g\\ =271.\end{array}$$

$$7d+28f+84g.$$

$b+7d+28f+84g=$ Cube of 10.
$27+133+336+504=1000.$

In the Biquadratick Power, I found after the same manner, that the *QQ.* of 3, that is, $81=b.$

$$81.$$
$$+7d+28f+84g+210h=QQ.\,10.$$
$$455+1400+3024+5040=10000.$$

Here $d=65.\ f=50.\ g=36.\ h=24.$

From

From whence I obſerved that the Figures joyned with the Differences, were all Figurate Numbers, and proceeding in order.

Thoſe joyned with the firſt Difference, *d*, to be Lateral Numbers.

Thoſe joyned with the ſecond Difference, *f*, to be Triangular Numbers.

Thoſe with the third Difference, *g*, to be Pyramidal Numbers.

Thoſe with the fourth Difference, *h*, to be Triangulo-Pyramidal Numbers.

Thoſe with the fifth Difference, *k*, to be Pyrami-pyramidal Numbers.

But as in Combinations, in caſe where there is no Figurate Table, or the Number of the Difference of the ſides do exceed the Table, the Powers may be thus found;

Take the Difference of the ſides of the Powers given and ſought, and increaſe it by an Unite as often as the Index of the Power is, ſave one, placing them with the Sign of Multiplication × between them for a Dividend; then place an Unite with the like Number of Figures increaſing by an Unite with the Sign of Multiplication × between them for a Diviſor; then prepare the Terms, by dividing the Dividend and Diviſor by each ſeveral Number in the Diviſor, and the Diviſor will be brought to an Unite; then multiply what is remaining in the *Dividend*, for a *Quotient*.

The *QQ.* of 10 is 10000; what is the *QQ.* of 20?

Where, $b=10000.\ d=3439.\ f=974.\ g=204.\ h=24.$
$20-10=10.$

$4\times3\times2\times1.\,)\,\frac{10\times11\times12\times13}{5\quad\quad 4}\,($

The Factors of the Quot.		*The Quot.*	*b.* 10000.
10		10 *d.* 10 × 3439.	34390.
5 × 11	=	55 *f.* 55 × 974.	53570.
5 × 11 × 4		220 *g.* 220 × 204.	44880.
5 × 11 × 13.		715 *h.* 715 × 24.	17160.

The *Q. Q.* of 20. 160000.

This differs from the manner of operation in Combinations in this, that there the Quantities do decreaſe, and here they do increaſe in Arithmetical Progreſſion.

The *Q.C.* of 5 is 3125 = *b.* 2101 = *d*, the firſt Difference 31; 1320 = *f.* 750 = *g.* 360 = *h.* 120 = *k.* what is the *Q.C.* of 105. 105 − 5 = 100.

5 × 4 × 3 × 2 × 1) 100 × 101 × 102 × 103 × 104 (

The Factors in the Quot.

100		100.
50 × 101	=	5050.
50 × 101 × 34		171700.
25 × 101 × 17 × 103	=	4421275.
5 × 101 × 17 × 103 × 104		91962520.

that is, *b.*		3125.
100 *d.* i.e. 100 × 2101	=	210100.
5050 *f.* i.e 5050 × 1320		6666000.
171700 *g.* i.e. 171700 × 750		128775000.
4421275 *h.* i.e. 4421275 × 360.		1591659000.
91962520 *k.* i.e. 91962520 × 120.		11035502400.

12762815625 = *Q.C.* 105.

This

This way shews how any Figurate Number for any Latus may be found, and any Figurate Table may be examined; as what are the Figurate Numbers for 30.

$$11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1 \,)\, 30\times31\times32\times33\times34\times35\times36\times37\times38\times39\times40.$$

	The Factors of the Quotients are	*The Quotients.*
1	1	1
2	30	30
3	15×31	465
4	5×31×32	4960
5	5×31×8×33	40920
6	31×8×33×34	278256
7	31×8×11×17×35	1623160
8	31×8×11×17×5×36	8347680
9	31×11×17×5×36×37	38608020
10	31×11×17×5×4×37×38	163011640
11	31×11×17×2×37×38×39	635745396
12	31×17×2×37×38×39×40	2311801440

Agreeing with my Figurate Table.

A Demonstration of the Rules in the 9th *and last Paragraphs.*

ALthough the Rule here given for the finding a Figurate Nuuber, and the Rule in the *9th.* Paragraph do differ in this, that the Quantities there do decrease, here increase in Arithmetical Progression, yet they do both spring from the last consequence cited by me out of Monsieur *Pascall* on Figurate Numbers.

For by that consequence any Figurate Number with its place being given, it is easie to find its next Number in a Diagonal Line either upward or downward.

As 165 being the *9th* Number in the *4th* Row, what is the *8th* Number in the *5th* Row? Say, as 4 is to 8, so is 165 to 330, the Number required.

What is the *10th* Number in the *3d* Row? Say, as 9 is to 3, so is 165 to 55, the Number desired.

3060 being in the *15th* Line in the *5th* Row, what are the Numbers joyinng to it in the same Diagonal Line? Say, as 5 . 14 :: 3060 . 8568. being in the *14th* Line in the *6th* Row. Again, 15 . 4 :: 3060 . 816. being the *16th* Number in the *4th* Row.

And consequently, the place of any Figurate Number being given, that Figurate Number may be thus found;

First, Find the Natural or Lateral Number in the same Diagonal Line, then the next Number to the Lateral in the same Diagonal Line, then the next to that, until you come to the Number desired.

The Lateral Number is known by substracting 2 from the Sum of both places; that is from the Number of the Rows and of the Lines.

As what is the *5th* Number in the *3d* Row?

The Sum of 5 and 3 is 8; from which substract 2, there remains 6, the Lateral Number of the same Diagonal Line. Then say, as 2 is to 5, so is 6 to 15, the Number required.

What is the *4th* Number in the *4th* Row? Say, $4+4=8$, lessened by 2, is equal to 6, its Lateral Number, then say as before, 2 . 5 :: 6 . 15. Again say, 3 . 4 :: 15 . 20, the Number sought for: Or thus;

$$2.6::5.\ \frac{6+5}{2}=15.\ \text{Again, } 3.4::\frac{6+5}{2}\ (15)\ \frac{6\times5\times4}{3+2}$$
$=20.$

What

What is the *6th* Number in the *6th* Row?

$6+6=12$, lessened by $2=10$, its Lateral Number, then find the next to its Lateral Number in the same Diagonal, and so continue working until you shall come to the Number required, thus; $2.9.10::\frac{10\times 9}{2}=45$, the *9th* Number in the *3d* Row.

Again, $3.8::\frac{10\times 9}{2}(45)\frac{10\times 9\times 8}{3\times 2}=120$. the *8th* number in the *4th* Row.

Again, $4.7::\frac{10\times 9\times 8}{3\times 2}(120)\frac{10\times 9\times 8\times 7}{4\times 3\times 2}=210$ the *7th* Number in the *5th* Row.

Again, $5.6::\frac{10\times 9\times 8\times 7}{4\times 3\times 2}(210)\frac{10\times 9\times 8\times 7\times 6}{5\times 4\times 3\times 2}=252$, the Number sought. The very Examples of the Rule delivered in the *9th* Paragraph to find 2, 3, 4, 5, and 6 Quantities in 10.

What are the *6th* Numbers in the *3d*, *4th*, and *5th* Rows?

These standing in several Diagonal Lines, must be wrought severally.

$6+3=9.-2=7$. its Lateral Number: $2.6::7.\frac{6\times 7}{2}=21$, the Number desired in the *3d* Row.

$6+4=10-2=8$, its Lateral Number: $2.8::7.\frac{7\times 8}{2}=28$. the *7th* Number in the *3d* Row.

Again, $3.6::\frac{7\times 8}{2}(28).\frac{6\times 7\times 8}{3\times 2}=56$, the Number required in the *4th* Row.

$6+5=11-2=9$, its Lateral Number, $2.9::8.\frac{8\times 9}{2}=36$, the *8th* Number in the *3d* Row.

Again,

Again, $3 . 7 :: \frac{8 \times 9}{2} (36) \frac{7 \times 8 \times 9}{3 \times 2} = 84$. the 7*th* Number in the 4*th* Row.

Again, $4 . 6 :: \frac{7 \times 8 \times 9}{3 \times 2} (84) \frac{6 \times 7 \times 8 \times 9}{4 \times 3 \times 2} = 126$, the Number desired in the 5*th* Row.

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 \\ 1 & 6 & \frac{6 \times 7}{2} = 21. & \frac{6 \times 7 \times 8}{3 \times 2} = 56. & \frac{6 \times 7 \times 8 \times 9}{4 \times 3 \times 2} = 126. \end{array}$$

6

$$\frac{6 \times 7 \times 8 \times 9 \times 10}{5 \times 4 \times 3 \times 2} = 252.$$

The Method of my General Rule to find any Figurate Number; then by dividing the *Divisor* and *Dividend* by each Number in the *Divisor*, the *Divisor* is brought to an *Unite*; 15 *Euc.* 5.

A

Here follows a Table of Figurate Numbers.

Units.	*Laterals.*	*Triangular.*	*Pyramidal.*	*Trianguli-pyramidal.*	*Pyrami-pyramidal.*
1	1	1	1	1	1
1	2	3	4	5	6
1	3	6	10	15	21
1	4	10	20	35	56
1	5	15	35	70	126
1	6	21	56	126	252
1	7	28	84	210	462
1	8	36	120	330	792
1	9	45	165	495	1287
1	10	55	220	715	2002
1	11	66	286	1001	3003
1	12	78	364	1365	4368
1	13	91	455	1820	6188
1	14	105	560	2380	8568
1	15	120	680	3060	11628
1	16	136	816	3876	15504
1	17	153	969	4845	20349
1	18	171	1140	5985	26334
1	19	190	1330	7315	33649
1	20	210	1540	8855	42504
1	21	231	1771	10626	53130
1	22	253	2024	12650	65780
1	23	276	2300	14950	80730
1	24	300	2600	17550	98280
1	25	325	2925	20475	118755
1	26	351	3276	23751	142506
1	27	378	3654	27405	169911
1	28	406	4060	31465	201376
1	29	435	4495	35960	237336
1	30	465	4960	40920	278256

I

1	1	1
7	8	9
28	36	45
84	120	165
210	330	495
462	792	1287
924	1716	3003
1716	3432	6435
3003	6435	12870
5005	11440	24310
8008	19448	43758
12376	31824	75582
18564	50388	125970
27132	77520	203490
38760	116280	319770
54264	170544	490314
74613	245157	735471
100947	346104	1081575
134596	480700	1562275
177100	657800	2220075
230130	888030	3108105
296010	1184040	4292115
376740	1560780	5852925
475020	2035800	7888725
593775	2629575	10568310
736281	3365856	13884156
906192	4272048	18156204
1107568	5379616	23535820
1344904	6724520	30260340
1623160	8347680	38608920

1	1	1
10	11	12
55	66	78
220	286	364
715	1001	1365
2002	3003	4368
5005	8008	12376
11440	19448	31824
24310	43758	75582
48620	92378	167960
92378	184756	352716
167960	352716	705492
293930	646646	1352078
497420	1144066	2496144
817190	1961256	4457400
1307504	3268760	7726160
2042975	5311735	13037895
3124550	8436285	21474180
4686825	13123110	34597290
6906900	20030010	54672300
10015005	80045015	84672315
14307150	44352165	129024480
20160075	64512240	193536720
28048800	92561040	286097760
38567100	131128140	417125900
52451256	183579396	600705296
70607460	254186856	854992152
94143280	348330136	1203322298
124403620	472733756	1676016044
163011640	635745396	2311801440

I

It is a chief property of these Figurate Numbers, That any Number in any Row is equal to the Sum of those in the preceding Row that stands even with and above it.

As 15 in the 3*d* Row = 5+4+3+2+1 in the second Row.

So 35 in the 4*th* Row = 15+10+6+3+1, in the third Row.

So 70 = in the 5*th* Row = 35+20+10+4+1, in the 4*th* Row.

So 126 = 70+35+15+5+1, &c. And therefore any Number in the Figurate Table is equal unto that above it, and to that even with it in the preceding Row; as 3 = 1+2. 10 = 4+6. 126 = 70+56. 1001 = 715+286. 462 = 252+210.

The Construction of the former Table.

Every Figurate Number is made by adding the Number preceding it to that above it; as in the 3*d* Row, 3 = 2+1. 15 = 5+10 66 = 11+55. In the 4*th* Row, 35 = 15+20. 220 = 55+165. 680 = 120+560. In the 7*th* Row. 28 = 21+7. 210 = 126+84. 5005 = 2002+3003. From whence Monsieur *Pascall* in a Treatise on this Subject, called *Triangular Arithmetick*, hath drawn 19 several Consequences; whereof these following are the choicest.

1. Every Figurate Number is equal unto the Number preceding it, and all above it in the same Row; as in the 4*th* Row, 20 = 10+6+3+1. in the 6*th* Row, 126 = 70+35+15+5+1. in the 7*th* 28 = 21+6+1.

2. Every Figurate Number is equal to the Number above it, and all preceding it in the same Line; as in the 3*d* Row, 15 = 10+4+1. 10 = 6+3+1. in the 6*th* Row, 21 = 6+5+4+3+2+1. 126 = 56+35+20+10+4+1.

3. If the Numbers of the Lines, and Rows of any Figurate are unequal, by interchanging the said Numbers, you will find the said Figurate in another place; as the 2*d* Number in the 6*th* Row, and the 6*th* Number in the 2*d* Row, are

equal to 6; so the 5*th* Number in the 3*d* Row, and the 3*d* Number in the 5*th* Row, are equal to 15.

4. The Figurate Numbers in every Diagonal Line are double the Figurate Numbers in the preceding Diagonal Line; as, 1+4+6+4+1, is double 1+3+3+1, which is double 1+2+1, and that is double 1+1.

5. Any two Figurate Numbers standing together in a Diagonal Line, are in proportion one to another, as their places are respectively distant from their proper Line of Unites inclusively, that is, accounting the place of the lower Number from the collateral Line of Unites, and the place of the upper Number from the Capital Line of Unites; as 5 in the Second Row, and 10 in the Third Row, are in proportion one to the other, as 2 and 4; for that 5 is in the Second Row, accounting from the collateral Line of Unites, and 10 is in the 4*th* Line, accounting from the upper Line of Units; so 35 in the 5*th* Row, and 21 in the 6*th* Row, are in proportion, as 5 to 3: So 56. 70::4.5. So 28. 56::3.6. So 330. 462::5.7. So 300.2300::3.23.

A

A Continuation of the former Figurate Table to 8 Rows only, beginning at 31, and ending at 100; in regard the other Four Rows would swell too much.

Units.	Late-rals.	Triangu-lar.	Pyrami-dal.	Trianguli-pyramidal.	Pyrami-py-ramidal.
1	31	496	5456	46376	324632
1	32	528	5984	52360	376992
1	33	561	6545	58905	435897
1	34	595	7140	66045	501942
1	35	630	7770	73815	575757
1	36	666	8436	82251	658008
1	37	703	9139	91390	749398
1	38	741	9880	101170	850668
1	39	780	10660	111930	962598
1	40	820	11480	123410	1086008
1	41	861	12341	13575	1221756
1	42	903	13244	148995	1370754
1	43	946	14190	163185	1533939
1	44	990	15180	178365	1712204
1	45	1035	16215	194580	1906884
1	46	1081	17296	211876	2118760
1	47	1128	18424	230300	2349060
1	48	1176	19600	249900	2596960
1	49	1225	20825	270725	2869685
1	50	1275	22100	292825	3162510
1	51	1326	23426	316251	3478761
1	52	1378	24084	241055	3819816
1	53	1431	26235	267290	4187106
1	54	1485	27720	395010	4582116
1	55	1540	29260	424270	5006386
1	56	1596	30856	455126	5461512
1	57	1653	32509	487635	5949147
1	58	1711	34220	521855	6471002
1	59	1770	35990	557845	7028847
1	60	1830	37820	595665	7624512

1947-

1947792	10295472
2324784	12620256
2760681	15380937
3262623	18643560
3638380	22481930
4496388	26978328
5245786	32224114
6096454	38320568
7059052	45379620
8145060	53524680
9366819	62891499
10737573	73629070
12271512	85900584
13983816	99884400
15890700	115775100
18009460	133784560
20358520	154143080
22957480	177100560
25827165	202927725
28989675	231917400
32468436	264385836
36288252	300674088
40475358	341149446
45057404	386206920
50063860	436270780
55525372	491796152
61474519	553270671
67945521	621216192
74974368	696190560
82598880	778789440

Units.	Laterals.	Triangular.	Pyramidal.	Trianguli-pyramidal.	Pyrami-pyramidal.
1	61	1891	39711	635376	8259888
1	62	1953	41664	677040	8936928
1	63	2016	43680	720720	9657648
1	64	2080	45760	766480	10424128
1	65	2145	47905	814365	11238513
1	66	2211	50116	864501	12103014
1	67	2278	52394	916895	13019909
1	68	2346	54740	971635	13991544
1	69	2415	57155	1028790	15020334
1	70	2485	59640	1088430	16108764
1	71	2556	62196	1150626	17259390
1	72	2628	64824	1215450	18474840
1	73	2701	67525	1282975	19757815
1	74	2775	70300	1353275	21111090
1	75	2850	73150	1426425	22537515
1	76	2926	76076	1502501	24040016
1	77	3003	76079	1581580	25621596
1	78	3081	82160	1663740	27285336
1	79	3160	85320	1749060	29034396
1	80	3240	88590	1837620	30672016
1	81	3321	91881	1929501	32801517
1	82	3403	95284	2024785	34826302
1	83	3486	98770	2123555	36949857
1	84	3570	102340	2225895	39175752
1	85	3655	105995	2331890	41507642
1	86	3741	109736	2441626	43949268
1	87	3828	113564	2555190	46504458
1	88	3916	117480	2672670	49177128
1	89	4005	121485	2794185	51971283
1	90	4095	125580	2891985	54891018

90858768		869648208
99795696		969443904
109453344		1078887248
119877472		1198774720
131115985		1329890705
143218999		1473109704
156338908		1629348612
170230452		1749579064
185250786		1984829850
201359550		2186189400
218618940		2404808340
237093780		2641902120
256851595		2897753715
277962685		3176716400
300500200		3477216600
324540216		3801756816
350161812		4151918628
374471480		4529365776
406481544		4935847320
437353560		5373200880
470155077		5843355957
504981379		6348337336
541931236		6890268572
581106988		7471375560
622614630		8093990190
666563898		8760554088
713068356		9473622444
762245484		10235867928
814216767		11050084695
869107785		11919192480

Units.	Laterals.	Triangular.	Pyramidal.	Trianguli-pyramidal.	Pyrami-pyramidal.
1	91	4186	129766	3049501	57940519
1	92	4278	134044	3183545	61124064
1	93	4371	139415	3321960	64446024
1	94	4465	142880	3464840	67910864
1	95	4560	147440	3612200	71523144
1	96	4656	152096	3764376	75287520
1	97	4753	156849	3921225	79208745
1	98	4851	161700	4082925	83921670
1	99	4950	166650	4249575	87541245
1	100	5050	171700	4421275	91962520

Examination.

$7*6*5*4*3*2*1)\ 100*101*102*103*104*105*106.$

	Factors of the Quote.		*Quotient.*
2	50*101		5050
3	50*101*34		171700
4	25*101*17*103	=	4421275
5	5*101*17*103*104		91962520
6	5*101*17*103*52*35		1609344100
7	5*101*17*103*52*5*106		24370067800

Being the Figurate Numbers of the $\left\{\begin{matrix}3\\4\\5\\6\\7\\8\end{matrix}\right\}$ Order of 190.

9270-

927048304		12846240784
988172368		13834413153
1052618392		14887031544
1120529256		16007560800
1192052400		17191612200
1267339920		18406953120
1346548665		19813501755
1429840335		21243342120
1517381580		22760723700
1609344100		24370067800

As the Table of Figurate Numbers may thus be examined, so if there be any mistake in the Calculation of the Tables, you may find what it will amount unto at any place required; for all Figures in the same Row or Line have the same Mistake: Those in the next Row or Line do proceed as natural Numbers; those in the third Row or Line inclusively; as *Triangulars:* those in the 4*th.* as *Pyramidals*; and so forward.

In the 6*th* Number in the 4*th* Row, suppose there is an Unite too much, 57, for 56, I would desire to know what such Mistake will amount unto in the 12*th.* Row, as also in the 100*th* Number in the 8*th* Row.

Take the Difference of the Rows, and the Difference of the Lines, and add an Unite to each, the Figurate Number belonging to that Row and Line is the Number desired; as,

$12-4=8+1=9.$ Row $\}$ The 25*th* Number in the 9*th*
$30-6=24+1=25.$ Line $\}$ Row, *viz.* 10518300 is the Number required; so that the 30*th* Number will be 2322319740, instead of 2311801440.

 In

In the other Question,

$100-6=94+1=95$. Line	The 95*th* Number in the 5*th*
$8-4=4+1=5$. Row	Row, *viz.* 3612280, is the

Excess that the 100*th* Number in the 8*th* Row will be more than it ought: If the Mistake be in the first place more than an Unite, you must multiply the Figurate found in that Number.

If any false Figurate Number be given, to find where the Error is, if it proceed only from the Mistake of an Unit; In the 10*th* Row and 25*th* Line, the Figurate is 38567100, and in a false Table it is 36744200, occasioned only by the mistake of an Unite; but where I would desire to know.

Take the Differences of the Figurates 177100, which you will find to be the 20*th* Number in the 7*th* Row. Then,

Take the Difference of the Rows, and Difference of the Lines, and add an Unit to them, you have the place of the Error.

$25-20=5+1=6$. Line	The Error is in the 6*th* Number
$10-7=3+1=4$. Row	in the 4*th* Row, *viz.* 57 for 56.

If there be but one Error committed, the Difference of the true and false Figurates is alwayes a Figurate or a Compound of it.

I find in Two Treatises, one Entituled *Cardanus Promotus*, another about *Cubick Equations*, of the most Learned Mr. *Thomas Baker*, Minister of *Bishop-Nympton* in *Devonshire*, my Worthy Friend, a Person endued with profound Skill in *Algebra*, &c. whose pains on those Arguments, are now at the Press, and will be succeeded by more of the same kind, a Table of new Figurates, where the 6*th* Number in the 5*th* Row is 182, in this 126. I do desire to know where the first Difference of the Tables is, from whence spring all the other Differences.

1,	2,	2,	2,	2,	2.
1,	3,	5,	7,	9,	11.
1,	4,	9,	16,	25,	36.
1,	5,	14,	30,	55,	91.
1,	6,	20,	50,	105,	196.
1,	7,	27,	77,	182,	378.

By the foregoing Rule, $182-126=56$, which is the 6*th* Number in the 4*th* Row.

6—

$6-6=0+1=1$. Line } So that the first Difference is the
$5-4=1+1=2$. Row } first Number in the 2d Row, there being 2 instead of 1.

By the common Table of Figurates, according to the foregoing Rule, this may be made by adding to any Figurate the Number preceding it in the same Line. As,
What is the 4th Number in the 3d Row? I say, $10+4=14$.
What is the 5th Number in the 4th Row? I say, $35+15=50$.
$252+126=378$, the 6th Number in the 6th Row: $210+55=275$. $=10$th Number in the 4th Row.

APPENDIX.

To the End of Pag. 21, *may be further added,*

THat the Sum of all the Chances on 2, 3, or more Dice, and the Chances on the Regular Bodies, *viz.* the *Tetrahedron*, *Octahedron*, *Dodecahedron*, and *Icosahedron*, being marked on their sides as Dice, not exceeding the number of their sides, are Figurate Numbers, and if the Sum of their Chances do exceed their sides, then they are not Figurates; they beginning at an *Unit*, do proceed on until they do come to their middle Chance, then decreasing in the same manner, they do end in an *Unit*, whether it be on 2, 3, 4, or more Dice, or such Bodies: The Chances on 2 Dice, or 2 such Bodies, are natural Numbers; on 3 Dice, or 3 such Bodies, are triangular Numbers; on 4, are Pyramidal, and so forward.

So that in a *Tetrahedron*, the Sums of the 4 first Chances are Figurate Numbers; in a Cubick Die, the Sums of the six first Chances; in an *Octahedron*, the Sums of the 8 first; in a *Dodecahedron*, of the 12 first; and in an *Icosahedron*, the 20 first Chances are Figurates.

As in 3 *Tetrahedrons*, the Chances are 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
Their Sums are 1, 3, 6, 10, 12, 12, 10, 6, 3, 1.

The first and last 4 Figures of their Sums are Triangular Numbers; 12, which is the Sum of the Chances, 7, is not Figurate; for that $1+1+5=7$. and the *Tetrahedron* having but 4 sides, and consequently no 5, the Changes of $1+1+5=7$ are 3; which being substracted from the next Figurate in the same Row, *viz.* 15, rest 12; as before.

So

So on the 3 Cubick Dice,

The Chances are 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18.
Their Sum 1, 3, 6, 10, 15, 21, 25, 27, 27, 25, 21, 15, 10, 6, 3, 1

The first and last six are Triangulars; 25 and 27 are not Figurates, for that $1 + 1 + 7 = 9$ (there being not any 7 on any one Die) cannot be on the Dice, and consequently its Changes 3, must be deducted from the next Figurate 28, remains 25, as before. Again, $1 + 1 + 8 = 10$, and $1 + 2 + 7 = 10$, neither of which can be on the Dice, there being neither 7 nor 8, their Changes 3 and $6 = 9$, must be deducted from 36, the 8*th* Number in that Row; then you have 27, as before.

So on 4 Dice; the first and last six Chances are Pyramidal Numbers, *viz.* 1, 4, 10, 20, 35, 56; the other, 80, 104, 125, 140, and 146 are not on the same reason.

If they had been all Figurates, their Sum would have been $715 + 715 + 286 = 1716$, exceeding 1296, the Chances on 4 Dice by 420.

Upon a Re-examination of the *Table*, these Errors were found, which the Reader is desired to Correct thus.

Range 7, *Latus* 21, 230230. *R.* 9. *L.* 22. 4292145. *L.* 25. 10518300. *L.* 30. 38608020. *R.* 12. *L.* 12. 705432.

In the Continuation.

Range 4. *Lat.* 77. 79079. *L.* 80. 88560. *R.* 5. *L.* 38. 101270. *L.* 89. 2794155. *L.* 90. 2919735. *R.* 6. *L.* 41. 1221759. *L.* 48. 2598960. *L.* 80. 30872016. *R.* 7. *L.* 67. 156238908. *L.* 78. 377447148. *R.* 8. *L.* 63. 1078897248. *L.* 68. 1799579064. *L.* 95. 17199613200. *L.* 97. 1981350785.

Advertisement.

THere is newly Published an ingenious small Tract, Entituled, *Artificial Versifying*; which sheweth to any one of ordinary capacity, though he understands not one word of *Latin*, or what a Verse means, how, by the help of 6 Tables, to make hundreds of *Hexameter Verses*, which shall be true *Latin*, true Verse, and Sense. The Conceit whereof is, that each Table contains 9 several Words, placed in Letters under a handsom disguise; which are these:

Table.

1. *Impia, sordida, aspera, turbida, tristia, horrida, turpia, pessima, perfida.*
2. *Dicta, facta, fata, bella, jura, vota, verba, dona, damna.*
3. *Mihi, tibi, inquam, viro, malis, vides, reor, aliis, scio.*
4. *Prædicunt, procurant, monstrabunt, promittunt, portabunt, causabunt, concedunt, producunt, confirmant.*
5. *Sidera, somnia, pignora, fœdera, crimina, tempora, dogmata, jurgia, pocula.*
6. *Multa, quædam, certa, tantum, planè, sola, prava, semper, sæpe.*

So that if you take any one word out of each Line, you will have a true *Hexameter* Verse.

It will be required to know how many several Verses by these Lines may be made? I Answer, according to the fore going Rule, *pag.* 16. of *Composition of Quantities* (without taking any notice of the permutation of places, that may be; for you may change the places of all words in the first unto the 5*th* Line, and 5 words of the 6*th* into the second Line) there may be made 531441 Verses, it being the Cubo-cube of 9; namely,

ly, above 30 times as many Verses as are in *Virgil*; for in his Works Printed at *Cambridge*, 1632, there are 49 Pages, and each full Page contains 30 Verses, so that there are not in *Virgil* above 17712 Verses. For if you join the Words in the first Line with those in the second, you will have 81 several Combinations of two Words, the Square of 9; and if to those you add each Word in the third Line, you will have 729 Combinations of 3 Words, the Cube of 9: and if you proceed further,, you will find that each Line doth augment the former 9 times. Possibly you may conceive that 6 Words cannot be taken so many several wayes out of 54, the number of Words in the 6 Lines; according to the foregoing Rules, they may be taken 25827165 times, being the number of *Combinations* of 6 Quantities in 54.

The reason of the Difference may easily be discerned, for that you are not to take 2 Words in one Line.

And we further subjoin (as a Supplement to, or for the better understanding of the 18*th* Section) That the Variations of single Quantities are found by Multiplying the number of Quantities in all the Numbers that are under it, as 6, the Variations of 3 Quantities, is found by Multiplying 3 into 2, into 1. So 120, the Variations of 5 Quantities (or Changes on 5 Bells) is found by Multiplying 5 into 4, in 3, in 2 in, 1.

FINIS.

ERRATA.

PAge 4. line 23, for *and* 1, read *and* 4. p. 5. l. 14. for 54, r. 5×4. p. 6. l. 27. for 44 × 13, r. 44×43. p. 9. l. 11. for 2, r. ·. p. 10. l. 30, for 2 + 135, r. 21 + 35. p. 11. *l.* 7, Make the top-Figures thus; 1 2 3 4 5 6 7 8. p. 14. l. 10. for 19, r. 20. 8 + 28 + 56 + 70 + 56 + 28 + 5 + 1. *Ibid.* The Table of Variations ſhould have followed *Paragr.* 18. *p.* 13. *Ibid.* to the *9th* Variat. add 0. p. 18. l. 14. for *Sum*, r. *ſame.* p. 19. l. *ult.* for 3 Chang. r. 4. p. 20. l. 18. *Col.* 1. for 6, r. 4. after 2226, 4, inſert 2235, 12. *Ibid.* Caſt 14, for 1156. r. 1256. p. 21. *l.* 16. for 8, r. 1. l. 19 for 3, 4, r. 24. l 24. for *By* 5, 5. r. *By Sect.* 5. p. 23, l. 24. dele *and.* l. 30, for 4, 5, r. 3, 4, 5. p. 25. *penult.* for 220, r. 120. p. 26. l 10. for *no more*, r. *uſe more.* l. 30. for *b* = 3. r. *b* = 27. p. 27. l. 22. for 289, r. 28 *g*. p. 28. l. 11. for *b* + 6 *k* = ; r. *b* + 6 *k* = *b*, l. 13. for 660, 960. p. 34. *ult.* for 3+2, r. 3×2. p. 48. l. *ult.* for 190, r. 100, p. 49 l. 21. for *the* 12*th*. r. *the* 30*th Num. in the* 12*th*.

Relations of numbers

[illegible]

Robert